A non renewable resource extraction model and a discussion of its implications for water use in Yemen.
Firt draft, may still contain errors.

By Ekko van Ierland, Sanaa, 15th April 2006

How to decide about the optimal extraction of water over time?
We will use the famous Hotelling rule, which says that for a non renewable resource the price should increase every year with a percentage equal to the social rate of time preference.
In this paper I consider the stocks of fossil water in Yemen as a non-renewable resource. In addition to these stocks, also the annual net precipitation is available, but that is not the main topic of this paper.
Following Perman, Ma, McGilvray and Common (2003) imagine we have a resource stock, S_0, and we can extract a quantity of the resource R_t.
We have a demand for the resource as given in Figure 1.

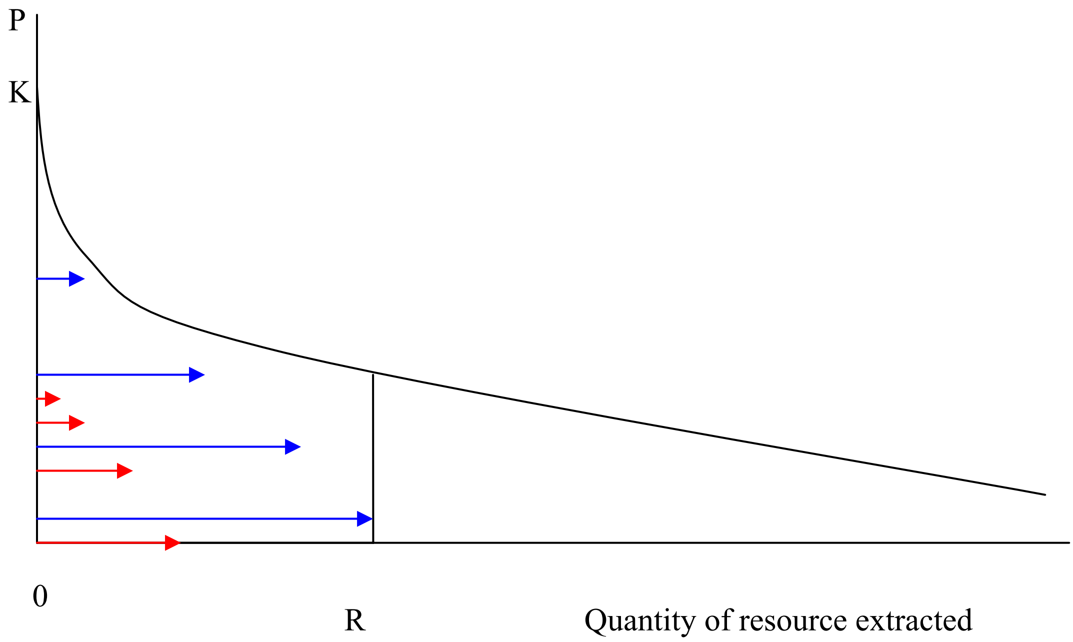

Figure 1: Resource extraction demand curve. K reflects the “choke price”, i.e. the price where nobody wants to use the commodity anymore.

The utility obtained in a year from resource extraction R (assuming for simplicity zero extraction costs) equals:

$$U(R) = \int_0^R P(R)dR$$

Differentiation with regard to R gives

$$\frac{\partial U}{\partial R} = P(R)$$

We can express discounted total utility (as measured if the full time horizon, till the resource is depleted)as

$$W = \int_0^T U(R)e^{-\rho t}dt$$

We want to make choices about how much we extract at each period of time, R_t, and the year when the stock will be depleted T, subject to the constraint that we cannot extract more than the stock that is available $\overline{S}$:

$$\int_0^T R_t dt = \overline{S}$$

Now we need some algebraical analysis to obtain the result. It is all basic algebra.

The remaining stock at time t, S_t is defined as:

$$S_t = \overline{S} - \int_0^t R_t dt$$

Differentiation with respect to time gives:

$\dot{S}_t = -R_t$, wich says that the change over time of the stock is equal to the extraction.

We then have the following maximiztion problem:

Max $W = \int_0^T U(R)e^{-\rho t}dt$

Subject to $\dot{S}_t = -R_t$

Optimization requires that the discounted marginal utility is equal at each point in time, that is

$\frac{\partial U}{\partial R} e^{-\rho t}$ = constant

Remember that the marginal utility of using the resource should equal the price in order to maximize welfare:

$$\frac{\partial U_t}{\partial R_t} = P_t$$

So for optimality we obtain:

$$\frac{\partial U}{\partial R} e^{-\rho t} = P_t e^{-\rho t} \text{=constant=} P_0$$

Rearranging this condition we obtain:

$P_t = P_0 e^{\rho t}$ or (after differentiation)

$$\frac{\dot{P}_t}{P_t} = \rho$$

This is the famous Hotellin g rule, which says that in order to exploit a non renewble resource in an optimal manner the price should increase with a percentage equal to the social discount rate of the social rate of time preference.

Optimal depletion requires that the stock will be zero in the final year and implies that extraction will be zero in the final period.

Note: for Yemen we need to be careful because it may be necessary to maintain a bufferstock in order to privde protection for years without rain.

This implies that the net price of the resource in the final period should equal the choke price. This is the price where the demand for the resource will go to zero and a substitute will be used, for instance desalinated water.

$$P_{\mathrm{T}} = K$$

How can we calculate what the time path of extraction should be?!

If $P_{\mathrm{T}} = K$ is combined with

$P_t = P_0 e^{\rho t}$, we obtain:

$$K_{\mathrm{T}} = P_0 e^{\rho T}$$

Now if we have a resource demand function as given in Figure 1 by

$P(R) = Ke^{-aR}$,

then we obtain:

$$P_t = P_0 e^{\rho t} = K e^{-aR}$$

This implies:

$$P_0 e^{\rho t} = P_0 e^{\rho \mathrm{T}} e^{-aR} = P_0 e^{-(aR - \rho \mathrm{T})}$$

$\rho t = -aR + \rho T$, and we obtain the required result.

$$R_t = \frac{\rho}{a}(T - t)$$

We now can obtain T in the following manner:

Recall

$$\int_0^{\mathrm{T}} R_t dt = \bar{S}$$

By substitution

$$\int_0^T \left[\frac{\rho}{a}(T - t) \right] dt = \bar{S}$$

$$\frac{\rho}{a}\left[Tt - \frac{t^2}{2} \right]_0^T = \bar{S}$$

$$\frac{1}{2}\frac{\rho}{a}T^2 = \bar{S}$$

or

$$T = \sqrt{\frac{2\bar{S}a}{\rho}}$$

This gives the initial royalty level

$$P_0 = K e^{-\rho T} = K e^{-\sqrt{2\rho \bar{S} a}}$$

Finally,

$$R_0 = \frac{\rho}{a}(T - 0) = \frac{\rho T}{a} = \sqrt{\frac{2\rho \bar{S}}{a}}$$

Let's take 2005 as the initial year of the study:

Water extraction from fossil stocks is estimated to be 1 billion m3. The stocks are estimated to be 50 billion m3, just as an example!!!!!.

Now imagine a = 1/(0.5*1)=2 and $\rho = 0.03$

Then $T = \sqrt{\frac{2\bar{S}a}{\rho}} = \sqrt{2 \bullet 50 \bullet 2 / 0.03 =} = \sqrt{200/0.03} = \sqrt{20000/3} \approx \sqrt{7000} \approx 83$

Then $R_0 = \frac{\rho}{a}(T-0) = \frac{\rho T}{a} = \sqrt{\frac{2\rho\bar{S}}{a}} = 0.03 \bullet 83/2 = 1.245$

$K = P_0 e^{aR_0} = 1 * e^{2*1.245} = 12$

$P_0 = Ke^{-\rho T} = Ke^{-\sqrt{2\rho\bar{S}a}} = 1.03$

$P_T = P_0 e^{\rho T} = 1.001 \bullet 2.7^{0.03 \bullet 83} = 12$

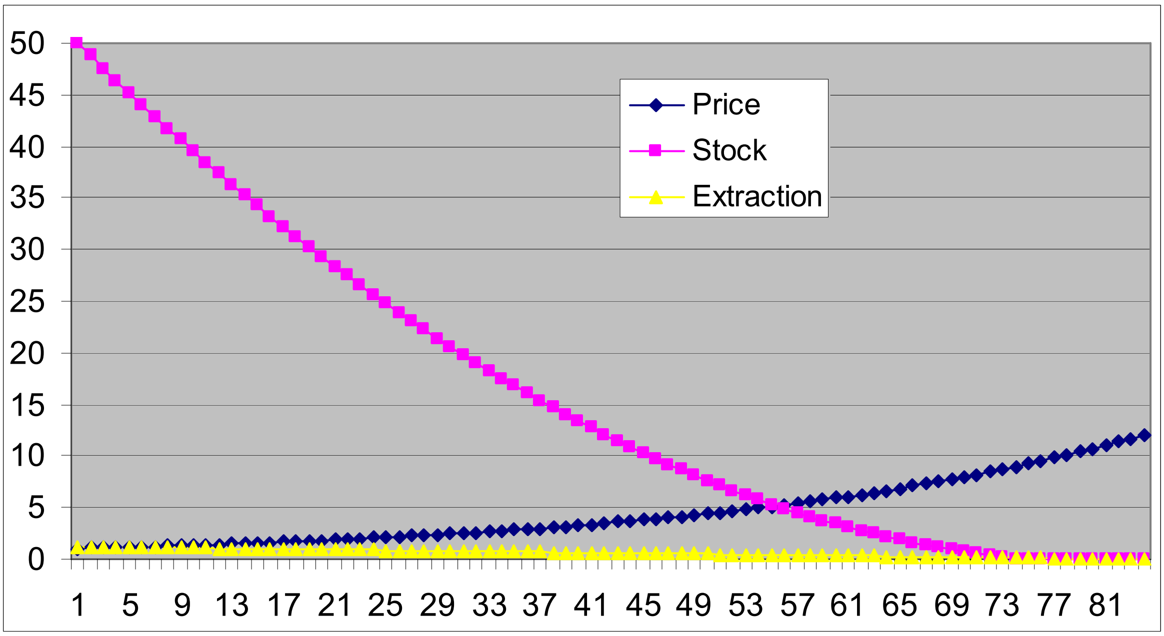

Figure 1: Development of stock, extraction and price over time.

If water would be a normal resource with proper substitutes, you could exploit it as a non-renewable, non-essential resource.

However, water is essential to Yemen, so you need to make sure that sufficient water will be available in the long run. This will require very careful water management and protection of the existing aquifers. They provide a top quality resource at very low costs.

In the meantime efficiency improvement is very important and better use of rain water also provides opportunities. Also the selection of crops is important because some crops use much more water than other crops.

In the longer run also additional water can be made available by means of desalination of sea water. It would be worthwhile to investigate the options for desalination by means of solar energy or in combination with generation of electricity as is being done in for instance Dubai.

However, my expectation is that water conservation in agriculture (and in qat production?!) could be more efficient.

The main thing is the availability of water on *a per capita* basis. Therefore it is also very important to reflect on what the carrying capacity of the environment is: how many people can be supported by the available natural resources that are available in Yemen?

GDP growth on per capita basis is also determined by the rate of population growth. At lower rates of population growth economic growth translates more directly into growth of per capita GDP.

In the longer run the price of water in the various regions should reflect the full cost of provision in that region, and the price of water should be the same for all sectors of the economy in that region.

However, if there are strong considerations of income distribution than some sectors may be supported by means of a lower water price, as a kind of subsidy. It would however be better to provide this income support in a different manner for example by means of a tax deduction, because than the scarcity of water is correctly reflected in its price and less inefficient use of water will take place.

In the longrun the water consumption cannot and will not exceed the quantity of water that is made available through recharge of the aquifers from the mountains, the direct use of rainwater and the use of recycled water, supplemented by water that may be imported from other regions (but at what price?) or water that is gained from desalination.

The key question is to avoid the application of water in sectors that provide hardly any value added or in sectors where it is used very inefficiently. This leads to the waste of water at the expense of the opportunities of providing low cost water in the future.

Basically the issue is how to make a smooth transition towards a situation where the use of water as a renewable resource is sustainable (includig reuse and recyling of water) and where additional water can be obtained from desalination, which is probably only possible at much higher water prices than the current price of water. It implies that the revenues of the use of the fossil water stocks need to be saved and reinvested in strengthening the competitive position of the economy of Yemen, in order to provide opportunities for next generations.